Problem 11.1

Given: $\vec{M} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{N} = 4\hat{i} + 5\hat{j} - 2\hat{k}$

Determine: $\vec{M}x\vec{N}$

The evaluation of a cross product in unit vector notation requires the evaluation of a matrix whose first row is comprised of unit vectors, whose second row lists the components of the first vector (in this case "M"), and whose third row lists the components of the second vector. There are a number of ways to do this evaluation. I prefer to reproduce the first and second columns on the right (you will see this done on the next page), then do the same cross multiplying over and over again. In any case, the bare bones matrix for this situation looks like:

$$\vec{M}x\vec{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ N_x & N_y & N_z \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 4 & 5 & -2 \end{vmatrix}$$

To hand-evaluate a matrix (versus using a calculator) is easy to do but not easy to explain. For the *x-component*, I've laid out the math below. Beyond that, I've simply finished off the problem. If you are confused about this, ask about it in class. For the $\hat{i}$ component of the cross product:

$$\vec{M}x\vec{N} = \left|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 4 & 5 & -2 \end{matrix}\right| \begin{matrix} \hat{i} & \hat{j} \\ 2 & -3 \\ 4 & 5 \end{matrix}$$

$$= (\hat{i})[(-3)(-2) - 1(5)] + \ldots$$

$$= (1)\hat{i} + \ldots$$

Doing the whole matrix produces:

$$\vec{M}x\vec{N} = \left|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 4 & 5 & -2 \end{matrix}\right| \begin{matrix} \hat{i} & \hat{j} \\ 2 & -3 \\ 4 & 5 \end{matrix}$$

$$= (\hat{i})[(-3)(-2) - 1(5)] + (\hat{j})[(1)(4) - (2)(-2)] + (\hat{k})[(2)(5) - (-3)(4)]$$

$$= (1)\hat{i} + (8)\hat{j} + (22)\hat{k}$$